where $C_1/\pi^2C_0 = -1$ for simply supported streamwise edges and -1.236 for streamwise edges, which are clamped.

3. Discussion

It follows from Eq. (7) for an infinite span panel ($a/b = 0$) and with $D \rightarrow 0$, that if $\bar{N}_x = -N_x$

$$\frac{2qa^3}{\beta} = \frac{4\pi^3}{3}\left[\frac{\bar{N}_xa^2}{\pi^2}\right]\left[\frac{\bar{N}_xa^2}{6\pi^2D}\right]^{1/2} \tag{8}$$

i.e.,

$$[2q/\beta\sigma_x][E^1/\sigma_x]^{1/2} = [2/3]^{3/2} \tag{9}$$

or

$$[2q/\beta] = [2/3]^{3/2}[\bar{N}_x{}^3/D]^{1/2} \tag{10}$$

Clearly, when $M \gg 1$, $\beta \rightarrow M$ and Eq. (9) becomes identical to Eq. (1). Thus, Eq. (6), which is derived in Ref. 4 from an exact analysis for large negative values of $\bar{A}$, leads in the limiting case of $D \rightarrow 0$ to the result obtained by an alternative procedure in Ref. 1.

For an infinite span plate, but with $D > 0$, Eq. (7) can be adapted to become

$$\left[\frac{2q}{\beta\sigma_x}\right]\left[\frac{E^1}{\sigma_x}\right]^{1/2} = \alpha^2 = \left[\frac{2}{3}\right]^{3/2} \times [1 + 10\pi^2\epsilon^2][1 + 4\pi^2\epsilon^2]^{1/2} \tag{11}$$

where $\epsilon^2 = [E^1/\sigma_x][h/a]^2$ and Eq. (11) can be shown to give the form of Fig. 2 in Ref. 1. Conversely, if $N_x = 0$, Eq. (7) becomes

$$\lambda_{cr}\bar{\epsilon}^3 = \bar{\alpha}^2 = [\tfrac{2}{3}]^{3/2}[1 + 10\pi^2\bar{\epsilon}^2][1 + 4\pi^2\bar{\epsilon}^2]^{1/2} \tag{12}$$

where $\bar{\epsilon}^2 = [C_0/2C_1](b^2/a^2)$. Thus in the new notation $\bar{\alpha},\bar{\epsilon}$, Eq. (12) has the same form as Eq. (11) for different initial assumptions. This was also shown in Ref. 2.

Reference 4 has shown that the solution, Eq. (6), from which Eqs. (7–12) have been obtained, is a very good approximation to the results for simply supported spanwise edges for small negative values of $\bar{A}$ but is less good for clamped edges. The corresponding exact results for all negative $\bar{A}$ as given in Ref. 4 have been considered in terms of the terminology $\bar{\alpha},\bar{\epsilon}$, and it is worth noting that the curves thus obtained for simply supported and clamped spanwise edges, respectively, agree with those presented in Fig. 1 of Ref. 2.

4. Conclusions

The exact analyses of Ref. 4 have been shown to agree with those of other references—in particular those of Refs. 1–3 for three-dimensional plates at high supersonic Mach numbers. For membrane-type plates in which $D \rightarrow 0$ and in the presence of large tensile chordwise in-plane stresses,

$$[2q/\beta] = [2/3]^{3/2}[\bar{N}_x{}^3/D]^{1/2} \tag{10}$$

This appears preferable to that given in Ref. 1 and Eq. (9), in that the quantities $\bar{N}_x$ and D are probably more easily defined than σ_x and E^1. This is so because $\bar{N}_x$ is dependent only on the external loading and only D is dependent on the material selected. For many membrane materials the determination of E^1 from the bending rigidity D would require an estimation of the effective thickness h, as would σ_x from $\bar{N}_x$. For a given material and flight condition, the required value of $\bar{N}_x$ to ensure stability is given from Eq. (10) by

$$\bar{N}_x = \tfrac{3}{2}D^{1/3}[2q/\beta]^{2/3} \tag{13}$$

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Influence Coefficients for Pressurized Cylindrical Shells of Finite Length

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In this Note the influence coefficients for thin-walled cylindrical shells of finite length subjected to uniform internal pressure and axisymmetric edge loads are presented. Only the final results are given here; the details of the derivation may be found in Ref. 1. Both cases, in which the edge loads are symmetric or antisymmetric with respect to the central cross-sectional plane of the cylindrical shell are considered. The nonlinear coupling effect of the pressure loading and edge loads is taken into account.

The equations governing the stresses and deformations of a cylindrical shell under internal pressure and axisymmetric edge loads are those given by Nachbar.[2] In the derivation of the equations, the secondary state direct stresses due to the axisymmetric edge loads are assumed to be a small perturbation on the primary state direct stresses or membrane state of stress due to the uniform internal pressure.

With the usual notations (Figs. 1 and 2) and with the following definitions

$$\xi = \frac{x}{(ah)^{1/2}}[12(1-\nu^2)]^{1/4};\quad \lambda = \frac{l}{(ah)^{1/2}}[12(1-\nu^2)]^{1/4};$$
$$w = \frac{w_2}{a};\quad \rho = \frac{pa}{2h\sigma_c};\quad \sigma_c = \frac{E}{[3(1-\nu^2)]^{1/2}}\frac{h}{a};$$
$$H = \frac{2H^*}{h\sigma_c}\left[\frac{\sigma_c}{2E}\right]^{1/2};\quad M = \left(\frac{M^*a}{D}\right)\left(\frac{\sigma_c}{2E}\right); \tag{1}$$
$$D = Eh^3/[12(1-\nu^2)]$$

where a is the radius, $2l$ is the length and h is the thickness of the cylindrical shell; the governing differential equation can be written as

$$(d^4w/d\xi^4) - 2\rho(d^2w/d\xi^2) + w = 0 \tag{2}$$

where ρ is the pressurization parameter defined in Eq. (1).

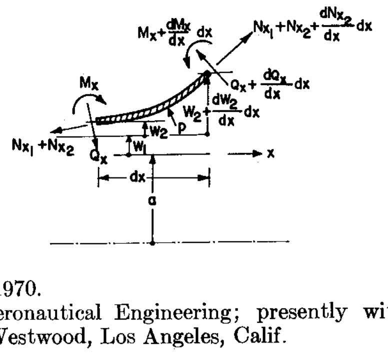

Fig. 1 Sign convention and forces on an element.

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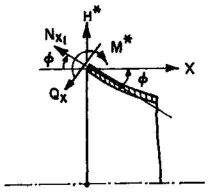

Fig. 2 Sign convention for edge loads H^* and M^*.

The boundary conditions to be satisfied at an edge of the shell are

$$H = -[d^3w/d\xi^3 - 2\rho dw/d\xi], \quad M = d^2w/d\xi^2 \tag{3}$$

where H and M are the nondimensional counterparts of the axisymmetric edge loads H^* and M^* applied at an edge of the shell as shown in Fig. 2.

Influence Coefficients

Symmetric loading

Consider the case of a cylindrical shell of finite length under internal pressure with axisymmetric edge loads that are symmetric with respect to the central cross-sectional plane of the shell as shown in Fig. 3. Then, writing for the deflection and slope at the edge of the shell as follows:

$$w(\lambda) = C_{11}^s H + C_{12}^s M, \quad \left(\frac{dw}{d\xi}\right)_{\xi=\lambda} = C_{21}^s H + C_{22}^s M \tag{4}$$

One obtains the following expressions for the influence coefficients C_{11}^s, C_{12}^s, C_{21}^s and C_{22}^s (where the superscript s stands for symmetric loading):

for $\rho < 1$:

$$C_{11}^s = \frac{2\alpha\beta[\cosh 2\alpha\lambda + \cos 2\beta\lambda]}{[\alpha(1-2\rho)\sin 2\beta\lambda + \beta(1+2\rho)\sinh 2\alpha\lambda]} \tag{5a}$$

$$C_{12}^s = C_{21}^s = \frac{[\beta \sinh 2\alpha\lambda - \alpha \sin 2\beta\lambda]}{[\alpha(1-2\rho)\sin 2\beta\lambda + \beta(1+2\rho)\sinh 2\alpha\lambda]} \tag{5b}$$

$$C_{22}^s = \frac{2\alpha\beta[\cosh 2\alpha\lambda - \cos 2\beta\lambda]}{[\alpha(1-2\rho)\sin 2\beta\lambda + \beta(1+2\rho)\sinh 2\alpha\lambda]} \tag{5c}$$

for $\rho = 1$:

$$C_{11}^s = \frac{2\alpha[\cosh 2\alpha\lambda + 1]}{[(1+2\rho)\sinh 2\alpha\lambda + 2\alpha\lambda(1-2\rho)]} \tag{6a}$$

$$C_{12}^s = C_{21}^s = \frac{[\sinh 2\alpha\lambda - 2\alpha\lambda]}{[(1+2\rho)\sinh 2\alpha\lambda + 2\alpha\lambda(1-2\rho)]} \tag{6b}$$

$$C_{22}^s = \frac{2\alpha[\cosh 2\alpha\lambda - 1]}{[(1+2\rho)\sinh 2\alpha\lambda + 2\alpha\lambda(1-2\rho)]} \tag{6c}$$

for $\rho > 1$:

$$C_{11}^s = \frac{2\alpha\delta[\cosh 2\alpha\lambda + \cosh 2\delta\lambda]}{[\delta(1+2\rho)\sinh 2\alpha\lambda + \alpha(1-2\rho)\sinh 2\delta\lambda]} \tag{7a}$$

$$C_{12}^s = C_{21}^s = \frac{[\delta \sinh 2\alpha\lambda - \alpha \sinh 2\delta\lambda]}{[\delta(1+2\rho)\sinh 2\alpha\lambda + \alpha(1-2\rho)\sinh 2\delta\lambda]} \tag{7b}$$

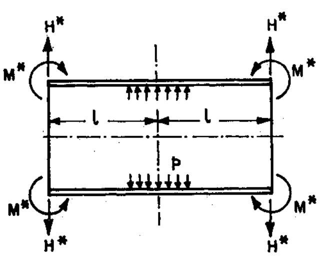

Fig. 3 Symmetric loading.

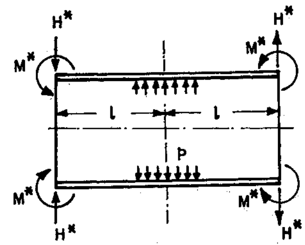

Fig. 4 Anti-symmetric loading.

$$C_{22}^s = \frac{2\alpha\delta[\cosh 2\alpha\lambda - \cosh 2\delta\lambda]}{[\delta(1+2\rho)\sinh 2\alpha\lambda + \alpha(1-2\rho)\sinh 2\delta\lambda]} \tag{7c}$$

where

$$\alpha = \left[\frac{1+\rho}{2}\right]^{1/2}; \quad \beta = \left[\frac{1-\rho}{2}\right]^{1/2}; \quad \delta = \left[\frac{\rho-1}{2}\right]^{1/2} \tag{8}$$

and λ is the nondimensional length parameter defined in Eq. (1).

Antisymmetric loading

When the axisymmetric edge loads are antisymmetric with respect to the central cross-sectional plane of the shell as shown in Fig. 4, writing

$$w(\lambda) = C_{11}^a H + C_{12}^a M, \quad \left(\frac{dw}{d\xi}\right)_{\xi=\lambda} = C_{21}^a H + C_{22}^a M \tag{9}$$

where the superscript a stands for antisymmetric loading, one obtains the following expressions for the influence coefficients C_{11}^a, C_{12}^a, C_{21}^a and C_{22}^a:

for $\rho < 1$:

$$C_{11}^a = \frac{2\alpha\beta[\cosh 2\alpha\lambda - \cos 2\beta\lambda]}{[\beta(1+2\rho)\sinh 2\alpha\lambda - \alpha(1-2\rho)\sin 2\beta\lambda]} \tag{10a}$$

$$C_{12}^a = C_{21}^a = \frac{[\alpha \sin 2\beta\lambda + \beta \sinh 2\alpha\lambda]}{[\beta(1+2\rho)\sinh 2\alpha\lambda - \alpha(1-2\rho)\sin 2\beta\lambda]} \tag{10b}$$

$$C_{22}^a = \frac{2\alpha\beta[\cosh 2\alpha\lambda + \cos 2\beta\lambda]}{[\beta(1+2\rho)\sinh 2\alpha\lambda - \alpha(1-2\rho)\sin 2\beta\lambda]} \tag{10c}$$

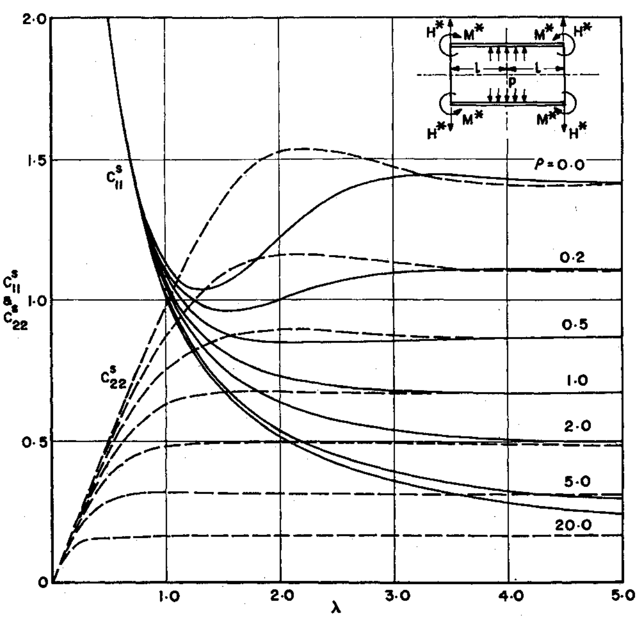

Fig. 5 Influence coefficients C_{11}^s and C_{22}^s for symmetric edge loads as functions of λ.

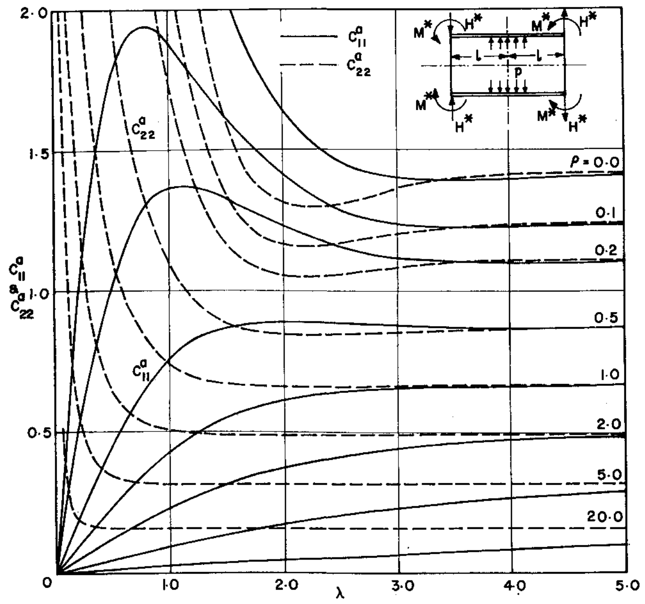

Fig. 6 Influence coefficients $C_{11}{}^a$ and $C_{22}{}^a$ for antisymmetric edge loads as functions of λ.

for $\rho = 1$:

$$C_{11}{}^a = \frac{2\alpha[\cosh 2\alpha\lambda - 1]}{[(1 + 2\rho)\sinh 2\alpha\lambda - 2\alpha\lambda(1 - 2\rho)]} \tag{11a}$$

$$C_{12}{}^a = C_{21}{}^a = \frac{[\sinh 2\alpha\lambda + 2\alpha\lambda]}{[(1 + 2\rho)\sinh 2\alpha\lambda - 2\alpha\lambda(1 - 2\rho)]} \tag{11b}$$

$$C_{22}{}^a = \frac{2\alpha[\cosh 2\alpha\lambda + 1]}{[(1 + 2\rho)\sinh 2\alpha\lambda - 2\alpha\lambda(1 - 2\rho)]} \tag{11c}$$

for $\rho > 1$:

$$C_{11}{}^a = \frac{2\alpha\delta[\cosh 2\alpha\lambda - \cosh 2\delta\lambda]}{[\delta(1 + 2\rho)\sinh 2\alpha\lambda - \alpha(1 - 2\rho)\sinh 2\delta\lambda]} \tag{12a}$$

$$C_{12}{}^a = C_{21}{}^a = \frac{[\alpha\sinh 2\delta\lambda + \delta\sinh 2\alpha\lambda]}{[\delta(1 + 2\rho)\sinh 2\alpha\lambda - \alpha(1 - 2\rho)\sinh 2\delta\lambda]} \tag{12b}$$

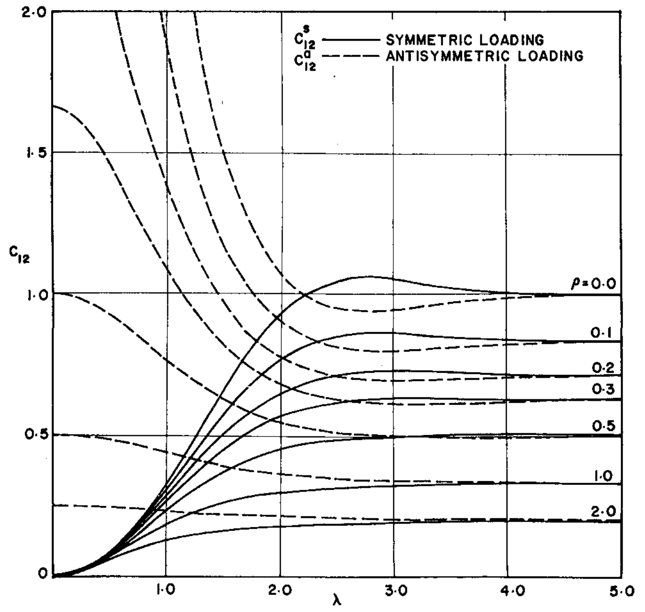

Fig. 7 Influence coefficient C_{12} for symmetric and antisymmetric edge loads as function of λ.

$$C_{22}{}^a = \frac{2\alpha\delta[\cosh 2\alpha\lambda + \cosh 2\delta\lambda]}{[\delta(1 + 2\rho)\sinh 2\alpha\lambda - \alpha(1 - 2\rho)\sinh 2\delta\lambda]} \tag{12c}$$

where α, β and δ are given by Eq. (8).

Numerical Results and Conclusions

Numerical calculations of the influence coefficients given by Eqs. (5–7) and (10–12) were done on a CDC 3600 computer for various values of the pressurization parameter ρ and length parameter λ. Figures 5, 6, and 7 show the graphs of the influence coefficients as a function of λ for various specified values of the parameter ρ, up to a value of $\lambda = 5.0$. Beyond this value of λ, the length effect becomes negligible and the values of the influence coefficients approach that for a semi-infinite cylindrical shell under axisymmetric loads at its edge.[2] For $\rho = 0$ the expressions for the influence coefficients given here agree with those given in Ref. 3 if due allowance is made for the sign conventions and notations.

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Free Vibrations of an Isotropic Nonhomogeneous Circular Plate

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Introduction

PAPERS pertaining to vibrations of the nonhomogeneous plate[1,2] or in different context, vibrations of plates of variable thickness[3,4] are not found in abundance in literature. Z. Mazuriewiez[1,2] in his two papers discussed the vibration of the nonhomogeneous rectangular plate. There the general problem was formulated in integral form and then reduced to an infinite system of linear homogeneous equations. The eigenvalues appear as roots of an infinite determinant and therefore the method does not afford exact values. This Note has discussed the transverse vibration of nonhomogeneous free circular plate. Nonhomogeneity of the plate is characterized by taking

$$E = E_0(1 - \rho^2)^\alpha, \quad \sigma = \sigma_0(1 - \rho^2)^{\alpha-2}, \quad (0 \leq \rho \leq 1)$$

where $\rho = r/a$, a the radius of the disc, E and σ are Young's Modulus and density, respectively, of the plate and α (integer > 3) is the index of nonhomogeneity. Explicit closed-form expressions are found for nodal frequencies, and effect of nonhomogeneity is shown in tabular form. The $\alpha = 3$ case coincides with Harris.[4]

Basic Equation

The equation of motion in polar coordinates for small deflection of plate[4] is

$$\sigma h \partial^2 W/\partial t^2 = (1 - \nu)\lozenge^4(D,W) - \nabla^2(D\nabla^2 W) \tag{1}$$

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